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Calculation of the Natural Vibrations of a Tapered Swept Back Rudder Fin

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A NOTE[1] recently published in the Journal of Aircraft presented an outline of a direct method for making calculations of generalized masses and stiffnesses of complex structures. Natural frequencies and modes of such structures can be computed by inserting these characteristic quantities into the equations for vibration analysis. An extensive application of this method to a tapered swept back rudder fin of a high-speed aircraft was made. A short survey of this application is presented and results shown can be compared with a relevant ground resonance test. The multicell fin was rigidly clamped at five spar-roots only, L1 to L5 as shown in Fig. 1.

As a first step, a set of assumed modes F_r, satisfying the conditions of the Rayleigh-Ritz method, must be chosen. A suitable Cartesian coordinate system was identified and the components of 51 modes were developed analytically as polynomials with three independent variables. In order to systematize the procedure, the following conceptual approach

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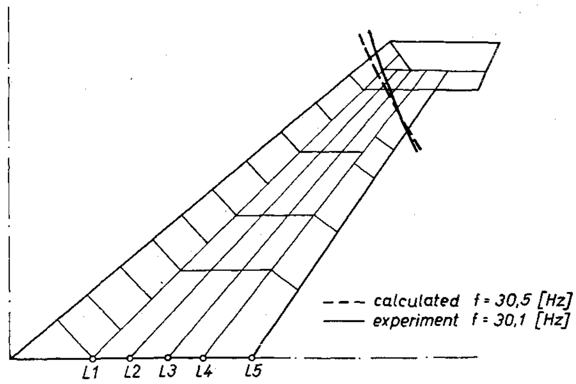

Fig. 1 The 2nd mode.

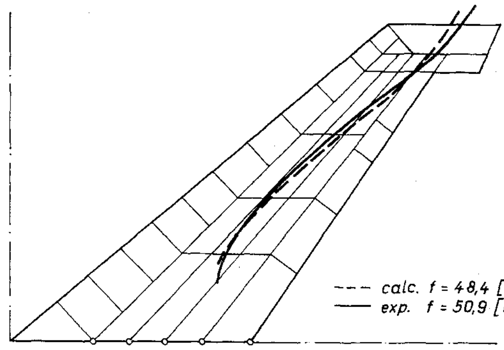

Fig. 2 The 3rd mode.

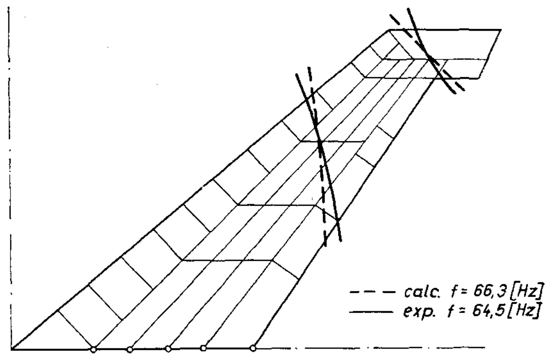

Fig. 3 The 4th mode.

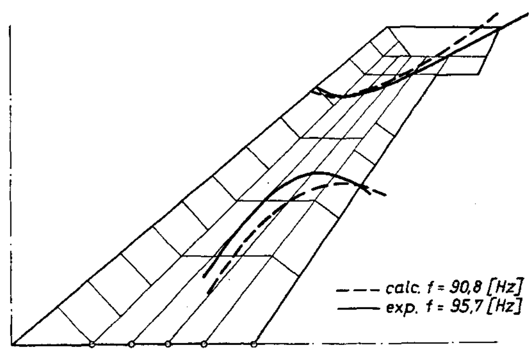

Fig. 4 The 5th mode.

Table 1 Comparison of measured and calculated natural frequencies[a]

No.	f_m	f_{51}	$\frac{100\cdot(f_{51}-f_m)}{f_m}$	f_{30}	$\frac{100\cdot(f_{30}-f_m)}{f_m}$	f_{20}	$\frac{100\cdot(f_{20}-f_m)}{f_m}$
1	8.8	9.0	+2.6	9.1	+3.1	9.2	+4.0
2	30.1	30.5	+1.3	31.0	+3.0	31.3	+4.0
3	50.9	48.4	−4.9	49.7	−2.4	49.8	−2.2
4	64.5	66.3	+2.8	68.2	+5.7	69.0	+7.0
5	95.7	90.8	−5.1	94.6	−1.2	94.7	−1.0
6	110.5	121.4	+9.9	126.6	+14.5	127.5	+15.4
7	168.3	168.1	−0.1	174.4	+3.6	174.8	+3.9

[a] f = natural frequency, Hz. Indices: m = measured; 51 = calculation with 51 assumed modes; 30 = calculation, shear-deformation deglected; 20 = calculation with bending modes only.

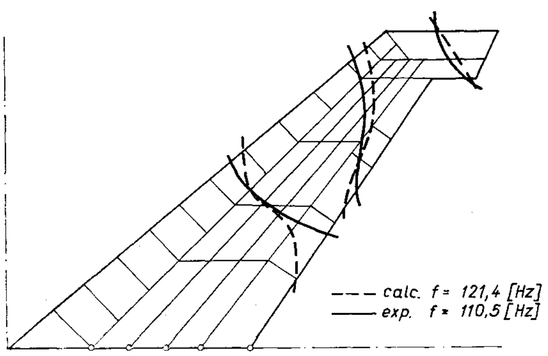

Fig. 5 The 6th mode.

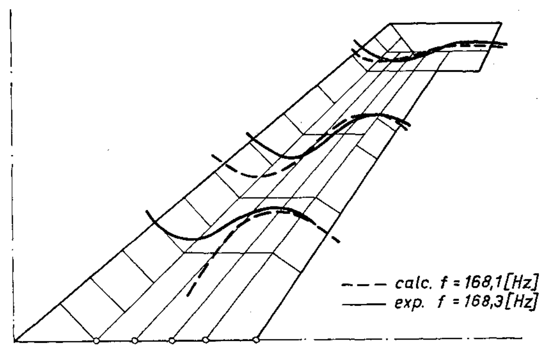

Fig. 6 The 7th mode.

was used. Twenty of the modes identify the displacement-strain relations of the plate bending theory. The components of these modes were constructed by the use of one dimensional Legendre-polynomials. Twenty-one of the modes took into account the shear deformations of the spar webs and the rib webs. These were obtained by an appropriate modification of the plate bending components. Finally, ten modes took into account the strain pattern at the discrete clamping points and may be regarded as a relaxation of the clamping effect due to the plate bending modes along the root of the fin in order to approximate the actual support conditions. These constraint modes decay quickly in spanwise direction, as would be expected from St. Venant's theory.

The fin structure was subdivided into a number of idealized elements. The skin was treated as an arrangement of plane triangles with constant thickness and the spar and rib webs were considered as tapered quadrilaterals. A two-dimensional stress pattern arising from the modes employed could be assumed in both elements. The flanges were considered as bars with linearly varying areas and axial stresses. In this context, there are no nodal points in the sense of the displacement method; however, the contours of the idealized elements can be considered as boundaries for an integration step over the volume.

To obtain the generalized stiffnesses φ_{rs} [see Eq. (1) in Ref. 1] and in an analogous manner, the generalized masses η_{rs}, the coefficients of the polynomials, and the structure data set were stored and automatically processed in an IBM 360/40 digital computer. In order to accomplish this, a suitable polynomial interpretation system was built up in FORTRAN IV language. An approximation method was developed to provide rapid integration over the volume of the structure.

Some calculated frequencies can be compared with the results of a ground resonance test in Table 1. The calculated and experimental nodal lines corresponding to the natural modes Nos. 2–7 are shown in Figs. 1–6. The calculated nodal lines stem from a computation including all 51 assumed modes. When the shear deformation modes were neglected, only natural modes (and as one can see from Table 1 also the frequencies) in excess of the fifth changed appreciably. This held true when in addition to neglecting shear deformation modes, the root constraint modes were also removed. Accordingly, the initial twenty plate bending modes remained for computation.

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Announcement: 1969 Author and Subjects Indexes

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